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The mean discordances of the above observations are $\pm''{\cdot}089$ for the Astrographic plates and $\pm''{\cdot}048$ for the Thompson plates. The probable error of a position of *Eros* derived from an Astrographic plate is $\pm''{\cdot}089 \times {\cdot}85 \times \sqrt{\frac{7}{6}} = \pm''{\cdot}082$, and for a Thompson plate $\pm''{\cdot}048 \times {\cdot}85 \times \sqrt{\frac{6}{5}} = \pm''{\cdot}045$. The result for the Thompson photographs is entirely satisfactory. Some of the photographs with the Astrographic telescope on October 26 are not very good, and it seems likely that the probable error would be smaller than $\pm''{\cdot}082$ on good nights. There is, however, an undoubted superiority of the Thompson photographs over those taken with the Astrographic equatorial.

As illustrating the accuracy which may be expected in the determination of the solar parallax from photographs of *Eros*, the following provisional results for October 26 and 27 may be of interest, but it is to be noted that they are not definitive, no correction for change of error in the ephemeris of *Eros* having been applied.

Instrument.	Mean Epochs G.M.T.						Deduced Solar Parallax.
		h	m		h	m	″
Astrographic	Oct. 26	6	58	Oct. 26	17	29	8·783
„	Oct. 26	17	29	Oct. 27	7	1	8·756
„	Oct. 27	7	1	Oct. 27	17	11	8·771
Thompson	Oct. 26	7	7	Oct. 26	17	17	8·798
„	Oct. 26	17	17	Oct. 27	7	34	8·805
„	Oct. 27	7	34	Oct. 27	17	13	8·763

Royal Observatory, Greenwich:
1904 *May.*

Note on the Optical Distortion of the Microscope of one of the Oxford Machines for Measuring Astronomical Photographs.

By H. C. Plummer, M.A.

The measures of the plates for the Oxford share of the Astrographic Chart Catalogue being now finished, a beginning has recently been made at the University Observatory in measuring with a modified form of instrument the photographs of the planet *Eros* which were taken in the autumn of 1900. The higher degree of accuracy which is demanded by parallax work has revealed defects in the optical arrangement of the apparatus which call for some examination. A brief note on the matter may be useful in drawing attention to a source of error which is possibly of general occurrence.

The type of measuring machine which has been in use at Oxford was described in *Monthly Notices*, vol. lv. p. 102. The modification which has been added recently consists in the pro-

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vision of micrometer screws by means of which the graduated glass diaphragm in the eyepiece of the microscope can be moved parallel to the two perpendicular scales.

The objective of the microscope is a simple achromatic combination having an aperture of about 1·2 cm. and a focal length of nearly 5 cm. The distance between the plate and the scale is about 20 cm. and the magnification is 1·5, the réseau space being 0·5 cm. and 100 divisions of the scale being equal to 0·75 cm. The angular field which should be covered has thus a radius of $3\frac{1}{2}^\circ$. It is clearly of great importance that there should be sharp definition without distortion over this field. The detection of the distortion which exists in the Oxford machine was due to the marked variation in the measured "runs," which was found on examination not to be due to irregularities in the réseau. For instance, a pair of réseau lines were set successively on the positions ·025 and ·525 on the scale. Several settings for "runs" were made in the two positions, and gave as a result $-{\cdot}0005$ and $+{\cdot}0038$ in the two cases, representing $-0''{\cdot}15$ and $+1''{\cdot}14$ respectively. The conditions under which these measures were obtained were identical except as regards the position of the réseau lines on the scale.

A partial calibration of the scale by comparing its divisions with distances measured by the micrometer screws was enough to show that neither the scales nor the screws could be held responsible for errors of this magnitude. The Zeiss scale leaves nothing to be desired in point of accuracy or clearness. The screws, on which the precision of the measures depends only in a minor degree, are also quite satisfactory. The experiments were therefore continued on the supposition that the discrepancies were due to optical distortion alone.

Systematic measures were made of the "runs" for two particular lines of a 5 mm. réseau at distances of ·05 along the X scale. The scale was first carefully centred, and measures were made along the whole length of the scale. Then the scale was shifted by turning the micrometer screws through five revolutions. In this way it was possible to prolong the measures beyond the range of the scale in its central position. The measured "runs" were then plotted as ordinates with abscissæ corresponding to the positions of the réseau lines on the scale. No distinction was made between equal distances from the centre to right or left, as the object was to find a distortion symmetrical about the centre. The points so obtained were smoothed by a graphical interpolation. The curve to which they approximated was parabolic in form, and corresponded fairly closely to the formula

$$E = 0^{\mathrm{I}}{\cdot}012\,(s - 0{\cdot}5)^2 = 3''{\cdot}6\,f^2,$$

where E is the effect of distortion on the "runs" and $s = f + 0{\cdot}5$ is the scale reading of either end of the réseau line expressed as a fraction of the 100 scale-divisions which cover a réseau interval. Thus the effect, which vanishes when the scale is symmetrically

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placed with respect to the two réseau lines, reaches the value $0^{\mathrm{r}}{\cdot}0030 = 0''{\cdot}9$ at the extreme positions. If the effect is to be attributed to distortion symmetrical about the centre of the field, then that distortion must be represented by the formula

$$\mathrm{D} = 0^{\mathrm{r}}{\cdot}004\, s^3 = 1''{\cdot}2\, s^3.$$

The effect of the distortion is in the direction away from the centre of the field, so that the magnification increases in the outward direction.

It must be concluded then that distortion exists in the microscope now examined, and that it is of considerable amount. The effect of "runs" and distortion is that, instead of the true reading s, we obtain the reading*

$$r_1 = s + as + ds^3$$

for the preceding réseau line, where d is the distortion coefficient and a the value which the "runs" would have if there were no distortion. The reading for the following réseau line will be

$$r_2 = 1 - [1 - s + a(1-s) + d(1-s)^3],$$

whence the measured "runs" are found to be

$$r_1 - r_2 = a + d(1 - 3s + 3s^2).$$

The mean "runs" derived from the measures will therefore be $a + \frac{1}{2}d$, while the "runs" corresponding to $s = 0{\cdot}5$ will be $a + \frac{1}{4}d$. There are then three ways which suggest themselves of correcting the reading r_1: (a) by following the usual method and adopting the value $(1-s)r_1 + sr_2$; (b) by correcting r_1 for the mean "runs;" (c) by correcting r_1 for the "runs" corresponding to $s = 0{\cdot}5$. The effect of the distortion on the final measure will be in the three cases:

$$(a)\ ds(1-s)(2s-1)\ ;\ (b)\ \tfrac{1}{2}ds(2s^2-1)\ ;\ (c)\ \tfrac{1}{4}ds(4s^2-1).$$

These effects are here tabulated, the unit being $0^{\mathrm{r}}{\cdot}0001 = 0''{\cdot}03$, with the value of d given above:

s	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
(a)	0	−3	−4	−3	−2	0	+2	+3	+4	+3	0
(b)	0	−2	−4	−5	−5	−5	−2	0	+4	+11	+20
(c)	0	−1	−2	−2	−1	0	+2	+7	+12	+20	+30

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* Since symmetrical distortion alone is looked for, no term bs^2 is included. But such a term might be produced by a sensible tilt of the scale. This effect has been noticed in the case of the Cambridge machine, and is remedied by mechanical readjustment. It may be noticed that the effect of terms depending on even powers of s will be eliminated by measuring in direct and reversed positions of the plate, while the effect of radial distortion will not be so eliminated.

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With the method (*a*), usually adopted for weighting the readings of the two réseau lines, the effect of distortion is much reduced, and its influence on the measures will scarcely be appreciable when these are only made to $0^{\mathrm{r}}\cdot001$. But in more accurate work the defect is more serious. If a method like (*b*) or (*c*) be adopted, in which the reading of one réseau line alone is corrected for "runs," these being taken to be constant, it will always be possible to choose the nearer line to the measured object. Hence no attention need be paid to the large numbers occurring in the second half of the above table, and it is clear that method (*c*) is better than method (*b*), and even preferable, so far as the effect of distortion is concerned, to the ordinary method. In method (*c*), by adopting the "runs" as measured in the centre of the scale, we use the minimum measured "runs." Even so the measures are clearly over-corrected, and if we seek further for the most advantageous value to use over the range $s = 0$ to $s = 0\cdot5$ we arrive at the number $a + \frac{1}{8}d$, for this will make the mean error over the range zero. The outstanding error in the final coordinate is given here with the same units as in the above table :

s	0·0	0·1	0·2	0·3	0·4	0·5
Error	0·0	−0·5	−0·6	−0·4	+0·6	+2·5

The distortion arises from want of flatness in the field, and to this again is due a marked absence of good definition at a distance from the centre. Any appreciable distortion within the field to be covered, even when not complicated by bad definition, is inconvenient as requiring corrections which might be obviated by better optical means. That this can be done may be inferred from the performance of photographic lenses. It may be necessary to use a doublet, as in the Cambridge instrument, or at least a triple lens. It is to raise this question that the present note has been written. Can objectives be obtained which are sensibly free from distortion,* and is it necessary to substitute for the single objective an optical combination of greater complexity ?

Since the adoption of the réseau as an essential aid in the work of measuring, the machines employed for measuring astronomical photographs seem to fall under three distinct classes :

(*a*) The perpendicular threads in the focus of the microscope are fixed during the measurement. Relative motions in perpendicular directions between the plate and the microscope are produced by micrometer screws, so that the star and the four sides of the containing réseau square can be brought under the microscope threads. The mechanical arrangement of the apparatus

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* The answer to this first question must be taken to be in the affirmative, for the Cambridge machine appears to be entirely free from distortion of an amount which can be detected. It is not so clear whether the capacity of a properly designed single objective is necessarily inadequate to the requirements.

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must be of a high order of perfection, and the cost of the instrument must therefore be considerable. Accuracy will then depend on the quality of the screws, and will be affected by their deterioration with continued use. On the other hand little depends on the quality of the microscope. All that is required of it is good definition at the centre of the field.

(*b*) The microscope and plate are fixed during the measurement of the stars within a réseau square, and the threads in the focus of the microscope are moved by micrometer screws. The dependence on the quality of the screws is the same as in the former case, while the mechanical requirements are probably easier to satisfy. The centre of the réseau square is brought to the centre of the field of the microscope, and good definition without sensible distortion is required over an area corresponding to the square. If fixed threads, forming a square in the focus of the microscope, be added, we have the form of instrument adopted by Sir David Gill for the astrographic work at the Cape Observatory. By bringing the image of the réseau square into approximate coincidence with the fixed square a certain rapidity of measurement is attained, though at some sacrifice of accuracy owing to the partial neglect of "runs."

(*c*) The microscope is provided with two rectangular crossed scales, ruled on a glass plate and placed in the focal plane. The star to be measured is brought to the intersection of the scales, and the positions of the containing réseau lines on the scales must be determined. As this can be done with considerable accuracy by direct estimation, and a rigid connection between the plate and microscope while the measure is made is the only mechanical requirement, a useful instrument of this class can be made in the cheapest possible form (*British Association Report*, 1900, p. 676). Since a star may be anywhere within a réseau square, each arm of the crossed scale must have a length equal to a réseau space, and the microscope must give a field, free from distortion and of good definition, of a width equal to two réseau spaces.

The Oxford machine now examined belongs to this last type. The substitution of the graduated scale for the screw as the chief means of measurement is seen to increase the demand for optical efficiency in the microscope. Hence the importance of the question raised above as to the relative qualities of different types of objective. Perhaps it may be pointed out that the effective field required of the microscope may be reduced by extending the principle of the crossed scale and developing it into a reticle, *i.e.* a ruled réseau. If it were possible to rule such a reticle without sacrificing the clearness possessed by the simple scale the field might be reduced to little more than a single réseau space, while the rapidity of measuring all the stars within a single square might be increased.

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Note on the Influence of the Plate Constants on the Accuracy of the Position of an Object measured on a Photograph. By H. C. Plummer, M.A.

When a preliminary solution is made for the constants of a plate on which the corrected positions of all the stars are required, as in the case of plates for the Astrographic Chart catalogue, the manner of proceeding is sufficiently obvious. It is then simply necessary to employ as large a number of comparison stars as possible, and to use the most accurate positions which can be found for them. The case is, however, rather different when the object is to find, not a solution which is to be applied over the whole extent of the plate, but one which is to be used to correct the measured coordinates of a particular object. This is what is required in parallax work, for example. Here questions arise which concern the choice and number of the comparison stars, and it is important from the point of view of economy to decide the relative degree of accuracy with which their coordinates should be measured. Mr. Filon has discussed the matter very thoroughly * and his main inferences are probably well known. The subject seems sufficiently important to justify discussion in a slightly different way, even if little that is new be added.

The equations from which the plate constants are calculated are of the form

$$ax+by+c=\Delta x\,;\quad dx+ey+f=\Delta y$$

where Δx, Δy are the differences between the computed coordinates and the measured coordinates of the comparison stars, *i.e.* the projections on the axes of the distances Δr between the computed and the measured positions. It is assumed that the comparisons are of equal weight for both coordinates and for all the stars. The six-constant solution will be considered first. The normal equations are

$$\begin{aligned} a[x^2]+b[xy]+c[x]&=[x\Delta x]\,; & d[x^2]+e[xy]+f[x]&=[x\Delta y]\\ a[xy]+b[y^2]+c[y]&=[y\Delta x]\,; & d[xy]+e[y^2]+f[y]&=[y\Delta y]\\ a[x]+b[y]+cn&=[\Delta x]\,; & d[x]+e[y]+fn&=[\Delta y] \end{aligned}$$

where n is the number of comparison stars. But it is true, and formal proof might be given if necessary, that had the comparison been made according to any other axes the results would have been equivalent. Hence for theoretical purposes we may take as axes the principal axes of the momental ellipse at the centroid of particles of unit mass occupying the positions of the comparison

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* *Monthly Notices*, vol. lxii. p. 561.